Estimation of synchronous machine parameters by standstill tests

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Received 7 October 2008; received in revised form 29 March 2010; accepted 10 May 2010
Available online 19 May 2010

Abstract
In this paper we present the results of a time-domain identification procedure to estimate the linear parameters of a salient-pole synchronous machine at standstill.
A new approach is proposed for the estimation of synchronous machine coupled to DC-chopper and Pseudo Random Binary Sequences excitations; using data recorded during steady-state operation of the chopper-machine unit. This procedure consists of defining and conducting the standstill tests, identifying the model structure, estimating the corresponding parameters, and validating the resulting model. The signals used for identification are the different excitation voltages at standstill and the flowing current in different windings. We estimate the parameters of operational impedances, or in other terms, the reactances and the time constants.
The results are presented from tests on a synchronous machine of 3 kVA/220 V/1500 rpm.

Keywords: Synchronous machine; Parameter estimation; Standstill tests

1. Introduction
There are many suggested methods for the determination of synchronous machine parameters. Among the modern techniques used are neural networks, finite elements, on-line and off-line statistical methods, frequency response, load rejection and others [3,5,8,12–14].
Different measurement techniques, identification procedures and models structures are developed to obtain models as accurate predictors for the transient behaviour of generators.
The standstill modelling approach has received great emphasis due to its relatively simple testing method where the d- and q-axis are decoupled [2,6,10,14,15].
This paper deals with the experimental determination of synchronous machine parameters and models.
The characteristic parameters obtained from reduced models of the synchronous machine are proposed and there is no need to any hypothesis. The first order equations, which approximate all the behaviour of the machine, are given. These reduced models traduce the subtransient, the transient and the steady-state operations of the machine respectively.

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Depending on whether the synchronous machine is supplied with voltage, we propose different differential equations from which analytical expressions of the currents and the voltages are deduced. The expressions are given as functions of the characteristic parameters: open-circuit and short-circuit time constants and reactances.

As an application, we have worked on the following methods:

- DC decay test in armature winding at standstill.
- Excitation by DC-chopper.
- Excitation by PRBS.

The purpose of these studies is to determine the static experimental conditions to estimate the parameters of a salient-pole synchronous machine using DC decay, DC-chopper and Pseudo Random Binary Sequences excitations.

The tests are performed and studied on a synchronous machine \(S_n = 3 \text{ kVA}, U_n = 220 \text{ V}, I_n = 8 \text{ A}, N_n = 1500 \text{ rpm}\).

2. Synchronous machine modelling

The study of the electric machines based on the Park’s transform was already treated in several works and specialized publications. This model allows, by a change of reference frame, to pass from the stator system to the rotor system with elimination of certain variables.

The basic model consists to considering one salient-pole synchronous machine with one pair of poles to the rotor and a three-phase stator winding.

The field winding is on the rotor of the machine according to the axis of the salience (direct axis).

The presence of grid or cage of dampers to the rotor, are modeled by two equivalent circuits dampers; one on the direct axis and the other on the quadrature axis.
The six windings representing Fig. 1 are described by the following equations

\[
\begin{align*}
V_a &= R_a i_a + \frac{d\phi_a}{dt} \\
V_b &= R_a i_b + \frac{d\phi_b}{dt} \quad \text{armature (Stator)} \\
V_c &= R_a i_c + \frac{d\phi_c}{dt} \\
V_f &= R_i i_f + \frac{d\phi_f}{dt} \\
0 &= R_D i_D + \frac{d\phi_D}{dt} \\
0 &= R_Q i_Q + \frac{d\phi_Q}{dt}
\end{align*}
\]

(1)

The voltage applied to the D and Q circuits are null, since they are in short-circuit (Fig. 2).
2.1. Choice of model’s order

Using the Park’s $d$ and $q$-axis reference frame, the synchronous machine is supposed to be modelled with one damper winding for the $d$-axis and two windings for the $q$-axis ($2 \times 2$ model) as shown in Fig. 1 [1,7,8,14,15].

Damper circuits, especially those in the quadrature axis provide much of the damper torque. This particularly important in studies of small signal stability where conditions are examined about some operating point [13]. The second order direct axis models includes a differential leakage reactance. In certain situations for second order models, the identity of the transients field winding. Alternatively, the field circuit topology can alter by the presence of an excitation system, with its associated non-linear features.

By considering Fig. 3, the equations of the machine are:

- **Voltage equations**
  
  \[
  V_d(p) = r_a i_d(p) + p\varphi_d(p) + \omega r \varphi_q(p) 
  \]
  
  \[
  V_q(p) = r_a i_q(p) + p\varphi_q(p) - \omega r \varphi_d(p) 
  \]
  
  \[
  V_f(p) = r_f i_f(p) + p\varphi_f(p) 
  \]
  
  \[
  0 = r_D i_D(p) + p\varphi_D(p) 
  \]
  
  \[
  0 = r_Q i_Q(p) + p\varphi_Q(p) 
  \]

After eliminating $\varphi_f, i_f, \varphi_D, i_D, \varphi_Q, i_Q$ we obtains the following equations:

\[
V_d(p) = r_a i_d(p) + p\varphi_d(p) + \omega r \varphi_q(p) 
\]

\[
V_q(p) = r_a i_q(p) + p\varphi_q(p) - \omega r \varphi_d(p) 
\]

\[
\varphi_d(p) = X_d(p)i_d(p) + G(p)V_f(p) 
\]

\[
\varphi_q(p) = X_q(p)i_q(p) 
\]

This writing form of the equations of the machine has the advantage of being independent of the number of dampers considered on each axis.

In fact, it is the order of the functions $X_{d,q}(p)$ and $G(p)$, which depend on the number of dampers.

For a machine at standstill, the rotor speed is zero ($\omega = 0$) and using the $p$ Laplace operator, the voltage equations can then be written as:

- For the $d$-axis
  
  \[
  V_d = \left[ r_a + \frac{P}{\omega_0} X_d(p) \right] i_d + pG(p) V_f 
  \]
\[ V_t = \left[ r_t + \frac{P}{\omega_0} X_t(p) \right] i_t + \frac{P}{\omega_0} X_m i_d \]  

(13)

- For the \( q \)-axis

\[ V_q = \left[ r_q + \frac{P}{\omega_0} X_q(p) \right] i_q \]

(14)

with the operational reactances:

\[ X_{d,q,f}(p) = \frac{(1 + pT_{d,q,f}')(1 + pT_{d,q,f}'')}{(1 + pT_{d0,q0,f0}')(1 + pT_{d0,q0,f0}'')} \]

(15)

and the operational function \( G(p) \):

\[ G(p) = \frac{X_{md}}{r_t} \left( \frac{1}{1 + pT_{d0}} \right) \]

(16)

where \( d, q, f \) denote the \( d \)-axis, \( q \)-axis and field respectively. From these equations, it follows that only the three functions \( X_d(p), X_q(p) \) and \( G(p) \) are necessary to identify a synchronous machine.

The reduced operational admittances of the \( d \)-axis and \( q \)-axis are deduced from the input–output signals

\[ Y_{d,q}(p) = \frac{i_{d,q}(p)}{V_{d,q}(p)}, \]

or in other terms

\[ Y_{d,q}(p) = \frac{1 + p(T_{d0,q0} + T_{d0,q0}'') + p^2 T_{d0,q0}' T_{d0,q0}''}{r_a + p \left[ r_d(T_{d0,q0} + T_{d0}'') + \frac{3d}{\omega_0} \right] + p^2 \left[ r_q T_{d0} T_{d0}'' + \frac{3d}{\omega_0} (T_{d,q}' + T_{d,q}'') \right] + p^3 \frac{3d}{\omega_0} T_{d,q}' T_{d,q}''} \]

(17)

The reduced operational admittances take the following forms

\[ H_{d,q}(p) = \frac{b_0 + b_1 p + b_2 p^2}{1 + a_1 p + a_2 p^2 + a_3 p^3} \]

(18)

The problem lies in calculating the constants \( a_1, a_2, a_3, b_0, b_1 \) and \( b_2 \) by using non-linear programming methods. For this purpose we have used a program, which calculates the six parameters quoted above from the input–output signals for each axis. The structure of the model being selected, then the form of the transfer function is known, thus the order of the numerator and of the denominator of equation (18) are, for our case, \( n = 2 \) and \( m = 3 \).

The objective of our estimation task is to make the simulated model response matches that of the actual response by minimizing the error between the two of them. In order to minimize this error, a good optimisation technique is needed. For that we used a quadratic criterion to quantify the difference between the process and the model which is based on the Levenberg–Marquardt algorithm.

### 2.2. Levenberg–Marquardt’s algorithm

The Levenberg–Marquardt algorithm [11] is a general non-linear downhill minimisation algorithm. It dynamically mixes Gauss-Newton and gradient-descent iterations. The unknown parameters are represented by the vector \( x \), and let the noisy measurements of \( x \) be made:

\[ z(j) = h(j; x) + w(j), \quad j = 1, \ldots, k \]

(19)

where \( h(j) \) is a measurement function and \( w(j) \) is a zero-mean noise with covariance \( N(j) \). Since we are describing an iterative minimization algorithm, we shall assume that we have an estimate \( \hat{x}^- \) of \( x \). A new estimate \( \hat{x} \) maximizes

\[ \chi^2(\hat{x}) = \sum_{j=1}^{k} (z(j) - h(j; \hat{x}))^T N(j)^{-1} (z(j) - h(j; \hat{x})) \]

(20)
We form a quadratic approximation to \( \chi^2(\cdot) \) around \( \hat{x}^- \), and minimize this approximation to \( \chi^2(\cdot) \) to obtain a new estimate \( \hat{x}^+ \). In general we can write such a quadratic approximation as

\[
\chi^2(x) \approx \alpha - 2a^T(x - \hat{x}^-) + (x - \hat{x}^-)^T A(x - \hat{x}^-)
\]

for scalar \( \alpha \), done because these terms are generally much smaller and can in practice be omitted.

After differentiating we obtain \( \partial\chi^2/\partial x = -2a + 2A(x - \hat{x}^-) \), \( \partial^2\chi^2/\partial x^2 = 2A \).

At the minimum point, \( \hat{x} \) we have \( \partial\chi^2/\partial x = 0 \). This means that

\[
A(\hat{x}^+ - \hat{x}^-) = a. \quad (21)
\]

Thus we need to obtain \( a \) and \( A \) to compute the update. We now consider the form of \( \chi^2(\cdot) \) in (20). Writing the Jacobian of \( h(j, x) \) as \( H(j) = \partial h(j)/\partial x \), we have

\[
\frac{\partial\chi^2}{\partial x} = -2 \sum_{j=1}^{k} H(j)^T N(j)^{-1}(z(j) - h(j; x)), \quad (22)
\]

\[
\frac{\partial^2\chi^2}{\partial x^2} = 2 \sum_{j=1}^{k} H(j)^T N(j)^{-1}H(j) - 2 \sum_{j=1}^{k} \left( \frac{\partial h(j)}{\partial x} \right)^T N(j)^{-1}(z(j) - h(j; x)) \approx 2 \sum_{j=1}^{k} H(j)^T N(j)^{-1}H(j), \quad (23)
\]

In the last formula for \( \partial^2\chi^2/\partial x^2 \), the terms involving the second derivatives of \( h(j, \cdot) \) have been omitted. This is done because these terms are generally much smaller and can in practice be omitted.

Now we solve the above equations for \( a \) and \( A \) given the values of the function \( h(j) \) and the Jacobian \( H(j) \) evaluated at the previous estimate \( \hat{x}^- \). We have immediately \( A = \sum_{j=1}^{k} H(j)^T N(j)^{-1}H(j) \).

We now write the innovation vectors \( v(j) \) as

\[
v(j) = z(j) - h(j; \hat{x}^-)
\]

Then we have

\[
a = \sum_{j=1}^{k} H(j)^T N(j)^{-1}v(j) \quad (24)
\]

Combining Eqs. (21) and (24) we obtain the linear system

\[
A(\hat{x}^+ - \hat{x}^-) = a = \sum_{j=1}^{k} H(j)^T N(j)^{-1}v(j) \quad (25)
\]

To be solved for the adjustment \( \hat{x}^+ - \hat{x}^- \). The covariance of the state is \( P = A^{-1} \).

The update (25) may be repeated, substituting the new \( \hat{x}^+ \) as \( \hat{x}^- \), and improving the estimate until convergence is achieved according to some criterion. Levenberg–Marquardt modifies this updating procedure by adding a parameter \( \lambda \) to the diagonal elements of the linear system matrix before inverting it to obtain the update. \( \lambda \) is reduced if the last iteration gave an improved estimate, i.e. if \( \chi^2 \) was reduced, and increased if \( J \) increased, in which case the estimate of \( x \) is reset to the estimate before the last iteration.

3. Standstill tests

In this section, the practical aspects of measurements are described and synchronous machine conditions for standstill time-domain data are given [1,4,9,12].

Fig. 4 shows the experimental procedure of identification.

The alignment of the rotor can be accomplished with short-circuited field winding. A sinusoidal voltage is applied between two stator phases. The duration of the application of the voltage should be limited to avoid serious overheating of solid parts. The rotor is slowly rotated to find the angular positions corresponding to the maximum value of the excitation current that gives the direct axis and zero value of the excitation winding current that corresponds to the quadrature axis.
The machine is not saturated during standstill tests; in fact, the flux densities are below those on the more linear part of the permeability characteristic that is commonly referred to as “unsaturated”. The determination of the quantities referred to as the unsaturated state of the machine must be done from tests, with supply voltages (1–2%) of the nominal values.

3.1. Experimental procedure

The two principal characteristic parameters, which relate to the listed definitions, are:

- $Z_d(p)$: the direct axis operational impedance equal to $r_a + pl_d(p)$, where $r_a$ is the DC armature resistance per phase.
- $Z_q(p)$: the quadrature axis operational impedance equal to $r_a + pl_q(p)$.

The above two quantities are the stator driving point impedances. While the above quantities are consistent with the definitions, an alternative method of measuring theses parameters is given as follow:

$$G(p) = \frac{I_{fd}(p)}{pI_d(p)} \text{ for } E_{fd} = 0$$  \hspace{1cm} (26)

With a shorted field winding ($V_f = 0$), the $d$- and $q$-axis operational admittances are given:

$$Y_{d,q}(p) = \frac{i_{d,q}(p)}{v_{d,q}(p)} \text{ for } V_f = 0$$  \hspace{1cm} (27)
With a \(d\)-axis armature shorted \((V_d = 0)\), the field winding parameters can be obtained by:

\[
Y_f(p) = \frac{i_f(p)}{v_f(p)} \quad \text{for} \quad V_d = 0
\]  

(28)

The waveforms recorded for the various excitations used are shown in Figs. 5–13.
Fig. 8. $d$-Axis stator voltage and current for field winding shorted.

Fig. 9. $d$-Axis stator voltage and current for field winding open.

Fig. 10. Field voltage and current for stator winding shorted.
Fig. 11. $d$-Axis stator voltage and current for field winding shorted.

Fig. 12. $d$-Axis stator voltage and current for field winding open.

Fig. 13. Field voltage and current for stator winding shorted.
### Table 1
Identified synchronous machine parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DC-chopper</th>
<th>PRBS voltages</th>
<th>DC decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a$ (p.u.)</td>
<td>0.149</td>
<td>0.149</td>
<td>0.149</td>
</tr>
<tr>
<td>$R_f$ (p.u.)</td>
<td>4.95</td>
<td>4.95</td>
<td>4.95</td>
</tr>
<tr>
<td>$T_d'$ (s)</td>
<td>0.1856</td>
<td>0.1675</td>
<td>0.1840</td>
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<td>$T_d''$ (s)</td>
<td>0.0490</td>
<td>0.0526</td>
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<td>$T_{d0}'$ (s)</td>
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<td>$T_q'$ (s)</td>
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<td>$T_{q0}'$ (s)</td>
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<td>0.3758</td>
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<tr>
<td>$X_q$ (p.u.)</td>
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<td>1.5639</td>
<td>1.3880</td>
</tr>
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<td>$X_q'$ (p.u.)</td>
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<td>$X_q''$ (p.u.)</td>
<td>0.0396</td>
<td>0.0345</td>
<td>–</td>
</tr>
<tr>
<td>$X_{q''}$ (p.u.)</td>
<td>0.0218</td>
<td>0.0198</td>
<td>–</td>
</tr>
</tbody>
</table>

3.2. DC decay test

Figs. 5 and 6 show the current and voltage waveforms during the DC decay test, for the $d$-axis tests (field winding shorted and open, respectively). Fig. 7 shows field voltage and current for stator winding shorted.

3.3. Excitation by DC-chopper

In the same way, Figs. 8 and 9 show the current and voltage waveforms during the DC-chopper test, for the $d$-axis tests (field winding shorted and open respectively). Fig. 10 shows field voltage and current for stator winding shorted.

3.4. Excitation by PRBS

Figs. 11 and 12 show the current and voltage waveform during the Pseudo Random Binary Sequences excitation test, for the $d$-axis tests (field winding shorted and open, respectively). Fig. 13 shows field voltage and current for stator winding shorted.

4. Identification results

The parameters identified by using the various excitations signals are gathered in Table 1.

The time-domain approach offers a method, which can yield useful models, particularly in the data, is interpreted correctly. The proposed model, the quality and the experimental data used to identify the model parameters and the robustness of the estimation technique, affects the fidelity of synchronous machine models.

The results obtained with the various excitations and by using the same procedure of identification, show a good agreement between the various identified parameters. It should be noted that the various signals used made it possible to determine all the parameters of the equivalent circuits, except for the dc decay test which does not enable us to determine the parameters varying very fast.

5. Model validation

The identified $d$–$q$ axis models are verified by comparing their simulated $d$–$q$ axis stator currents responses against the measured standstill response, for that we present in Fig. 14, simulation results for $d$-axis stator current for signals among
those presented previously. The measured and simulated responses to off-line excitation disturbances comparison show that the machine linear parameters are accurately estimated to represent the machine at standstill conditions.

6. Conclusion

This paper presents a step-by-step procedure to identify the parameter values of the $d-q$ axis synchronous machine models using the standstill time-domain data analysis.

A three-phase salient-pole laboratory machine rated 3 kVA and 220 V is tested at standstill and its parameters are estimated. Both the model transfer function and the equivalent circuit model parameters are identified using the Levenberg–Marquardt algorithm.

The various excitations signals used gave very similar parameters results. Moreover, the simulation of measured and calculated parameters shows the validity of the results obtained. It should be noted that the various signals used
made it possible to determine all the parameters of the equivalent circuits, except for the DC decay test which does not enable us to determine the parameters very quickly varying.

Among the advantages claimed for the time-domain approach at standstill, is that the tests are safe and relatively inexpensive.

Furthermore, information about the quadrature axis, as well as the direct axis of the machine is obtained.

References